DESIGN OF OPEN SPANDREL REINFORCED CONCRETE ARCH BRIDGE

BY
R. J. JENSEN
E. O. MANDLER
C. H. MARX

ARMOUR INSTITUTE OF TECHNOLOGY

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Jensen, Raymond F.
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Of an Open Spandrel Reinforced Concrete Arch Bridge of Two Hundred and Ten Feet Span.

A Thesis

PRESENTED BY

RAYMOND F. JENSEN

EMIL O. MANDLER

CARL H. MARX

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Approved:

Alfred E. Phillips.
Prof. Civil Engineering

H. M. Raymond
Dean of Eng. Studies

L. C. Monin
Dean of Cultural Studies

DESIGN OF AN OPEN SPANDREL REINFORCED CONCRETE BRIDGE WITH SOLID RIB AND DIAPHRAGMS - 210' SPAN AND 44' RISE.

-METHOD OF DESIGN-

The method used in the design and analysis this arch was that given by Turneaure and Maurer in their treatise on "Principles of Reinforced Concrete." This application of the theory of the arch was followed throughout.

-THE ARCH RIB-

In the design of any bridge, certain assumptions must be made; this fact being more manifest in the case of a concrete or masonary arch rib. It is the usual custom to assume a preliminary design made by the aid of approximate or empirical rules or by reference to the proportions of existing arches. This arch is then analyzed and the results used in correcting the design, the corrected design may then inturn be analyzed, if it departs too greatly from the first assumed.

To begin with we agreed to make the arch rib solid instead of using separate arch rings, as has been the comman practise in most concrete bridges and viaducts. The spandrel were also made solid. These assumptions simplify the design somewhat as the dead load and live load were concentrated uniformly over the rib. The thicness of the rib t the crown was assumed 3 ft. and at the haunch as 5 ft. The roadway is supported on spandrel walls 18 inches thick resting upon the arch rib. The spacing of the diaphragms was assumed as 15 ft. making the distance between springing lines 210 ft. The rise of the arch at the crown was taken as 44'0". The spandrel walls at the crown is 5'0" fro the axis of the arch to the underside of the floor beam. The arch was designed with .5% of steel reinforcement above and below the axis.

NOTATION- (See Plate 1.)

Let H_o = thrust at the crown;

V_o = shear at the crown;

M_o = bending moment at the crown, assumed as positive when causing compression in the upper fibres;

N, V, & M = thrust, shear, and moment at any section;

R = Resultant pressure at any section resultant of N and V;

ds = length of a division of the arch ring measured along the arch axis;

n = number of divisions in one half of the arch;

I_a = moment of inertia of any section;

P = any load on the arch;

x, y = co-ordinates of any point on the arch axis referred to the crown as origin, and all to be considered as positive insign;

m = bending moment at any point in the cantilever, due to external loads.

Theoretically the gain in economy by the use of steel in the concrete arch is not great. If the pressure line is not depart from the middle third, the steel reinforces only in compression and in this respect is not as economical as concrete. If the line of pressure deviates farther from the center, resulting in tensile tresses in the steel, the conditions are such that these stresses must be provided for by use of the steel at very low working values. That is to say, the direct compression in the arch is so large a factor that the limiting stresses in the concrete will always result in very small unit tensile stresses in the steel where any tension exists at all.

Practically the value of reinfocement is very considerate. It renders an arch of much more secure and reliable structure, it greatly aids in preventing cracks due to any slight settlement, and by furnishing a form of construction of greater reliability makes possible the use of working stresses

Page 333- Turneaure and Maurer.

In the concrete considerably higher than is usual in plain masonary. Furthermore, in long spans such as ours, where the dead load constitutes by far the larger part of the load, any possible increase in average working stress counts greatly toward economy. It affects not only the arch but the abutment and foudation.

The roadway was made 30'0" from curb to curb, le ving room for two tracks for an electric railway. The roadway is to be paved with asphalt having a two inch cushion of sand. The sidewalks are 10'0" wide supported by cantilever brackets. The sidewalk is to be furnished with a concrete railing having a post at each panel point. An electric lamp is to be placed at every other post.

-DESIGN-

The analysis of any arch consists in the determination of forces acting at any section usually expressed as the thrust, the shear and the bending moment at that point.

The thrust we use, was taken to be the component of the resultant parallel to the arch, axis at the given point and the shear is the componen t at right angles to such axis. The thrust causes simple compressive tresses; the shear causes stresses similar to those produced by the vertical shear in a simple beam. The method of procedure will be to determine first, the thrust, shear, and bending moment at the crown. These being known, the values of similar quantities for any other section can readily be caloulated.

Before any of the above mentioned values can be determined the arch ring has to be divided into preliminary and final divisions, the central points of which we investigate for shear etc. In most cases the depth of the arch ring increases from crown towards springing line giving a variable moment of inertia. Considering the concrete only the moment of inertia will increase as d so that comparitively small change in depth will cause a large change in moment of inertia.

To maintain ds/I constant, the value of ds will therefore ne much greater near the springing line than at the crown and hence to secure the desired accuracy the length of division at the crown will need to be made fairly short. The value of ds/I to adopt so that there will be no factional division is: ds/I Si/n where

I is mean value of moment of inertia.

S is half lengths of the arch ring measured along the axis.

N is number of divisions in one half of the arch.

First we calculated the mean value of i for each divison of half the arch (see plate "A") after we had scaled the depths at the mid-points of each divison. Knowing the amount of steel in the arch we figured the moment of inertia of it about thesame axis. To get the total moment of inertia "I" we multiplied this value by 15 and added to it the I (Plate "A".) $I = I_a + 15\ I_s$

The average $i \Sigma = i/n = 3.4769/14$

The value of dsi being known, the proper length of ds for any part of the arch ring can readily be determined. The half length of the arch axis was found to be 116.97 feet. The first part of the table A relates to the preliminary 14 equal divisons. Each equal to 116.97/10 = 11.697 ft. The resulting values of i were ploted as shown in plate-3. The line ab is 116.97 feet long and was divided into 14 equal divisons as 1,2,3,etc. At the center of the several divisons the values of small i were laid off as ordinates i i i etc., and the curve cd was drawn through these points.

The area abcd $= 116.97/14 \times \Sigma i = 116.97 \times i_a$

This area is to be divided into fourteen equal parts each equal to dsi. Each of these parts will then be equal to $116.97 \times i_a/14 = 2.074$ as given below table A. Beginningat one end of this diagram the several equal areasare then laid off,

the values of i being scaled from the diagram and ds is equal to 2.074/i. These calculations are given in the latter part of table A, where are also given the values of I and d for the center points of the final subdivisions.

To obtain the thrust, shear and bending moment at the crown we used the formulaes:

$$H_o = \frac{n\Sigma my - \Sigma m\,\Sigma y}{2[(\Sigma y)^2 - n\Sigma y^2]}$$

$$V_o = \frac{\Sigma(m_R - m_L)x}{2\Sigma x^2}$$

$$M_o = \frac{\Sigma m + 2H_o\Sigma y}{2n}$$

In these equations the summations Σy, Σy^2, and Σx^2 are for one-half of the archonly; the summations Σm is for the entire arch and is equal to $\Sigma m_R + \Sigma m_L$; the summation $(m_R - m_L)x$ is a summationof the products $\Sigma(m_R - m_L)x$, in which m_R and m_L are the bending moments at corresponding points in the right and left halves which have equal abscissas x; and the summation Σmy is for the entire arch, but since symmetrical points have equal y's this quantity may be calculated as $\Sigma(m_R + m_L)y$

In designing an arch it is sufficient generally to determine the maximum stresses at the crown, the haunch, and the springing line. This will require sveral different positions of the live-load. For the crown the maximum positive moments are caused when a short length of the arch at the center (middle third) is loaded, and the maximum negative when the remaining portions are loaded. The maximum positive and negative moments at the haunch (about the 1/4 point) are caused when the whole span length is loaded. A condition was also taken with the half span length loaded.

The values of x and y in these equations, were accurately scaled from the drawing. The values of m and m were figured for the different loadings and their summation taken as shown in tables BC&D. A simple substitution in equations for H_0 V_0 and M_0 gave us the values for each of the three conditions of loading.

All the loads in this design were vertical, so that the graphical method might easily have been to advantage in determining the cantilever moments "m."

With these values calculated we lay out the force polygons using H as the pole distance and M/H as the distance above or below the axis. The equilibruim polygon was indrawn (plate C-2.) and the eccentric distances obtained. The thrust was measured or scaled directly from the true force polygon.

The total bening moment at any section, 1,2, 3, etc. was found from the equation :

$$M = m + M_o + H_o y \pm V_o x$$

The plus sign was used for the left half and the minus sign for the right half of the arch. Knowing the moment and thrust (Table E_1) at each point the eccentric distances were found since $e = M/H$. If calculations are to be made for more than one loading it will be noted that the denominators of the values for H_o, V_o, and M_o, do not change.

Having the values of the bending monents and ecoentricitiesat each of the fourteen points, the next step was to find the unit stresses in the concrete and steel. Tocalculate these stresses we used the formulae for simple beams:

$$\frac{M}{bh^2 f_c} = \frac{1}{12k}(1 + 24npa^2/h)$$

TO facilate the application, of this equation, Plates XIII andXIV pages 287 and 288 in Turneaure and Maurer" were used. Knowing the eccentricty anddepth of the beam, a simple division gave us e/h. In the first diagram, values of the eccentricities ,e/h, are given at the upper and lower margins; the ordinates from the lower margins to any curve are values of $(1 + 24npa^2/h)/12k$, and hence of $M/bh^2 f_c$, for the values p marked on that curve.

For instance take point nine on plate D

$e/h = .353/3.78 = .0745$ where $p = .5\%$

$M/bh^2f_c = 0.0685$ but $M = 46.4$

$\therefore f_c = 464 \times 12/\overline{3.78}^2 \times 12 \times 144 = 464$#/sq.in.

In this manner we figured the unit stress in the concrete for each point in the arch under the three loadings. This stress occurs in the upper fibres of the arch, while the value in the lower fibres is equal to :

$f_c' = f_c(1-1/k)$ which is always less than f_c.

The stresses in the steel were calculated from equations :

$f_s' = nf_c(1-d'/kh)$

$f_s = nf_c(1-d/kh)$

With the value of e/h we found the value of $1/k$ from figure 33, page 103 Turn. and Maurer.

By simple substitution and a little mathematics we obtained the stresses in the steel which as is shown in plate C were safe. Since all the stresses in the concrete and steel were under the allowable values of $f_c = 600$ and $f_s = 16000$ the arch issafe.

-TEMPERATURE STRESSES.-

The temperature stresses were obtained by means of the equation:

$$H_o = \frac{EI}{ds} \times \frac{ctln}{2\left[n\Sigma y-(\Sigma y)^2\right]}$$

where H is the thrust at the crown produced by the restraint of the abutment.

c = coefficient of expansion = .0000054

l = span 210'.

t = temperature in degrees = F = 30

E = coefficient of elasticity 1,500,000#/in.

I = moment of inertia ds/I 3.1

$$H_o = \frac{1500000x144}{3.1} \times \frac{.0000054x30x210x14}{2\left[14(3915.85)-\overline{129.61}^2\right]}$$

$$H_o = \frac{103000000}{148600} = 693.0\#$$

M_o = -693x129.6/14 = -6,418.9 ft. lbs.

The equilibruim polygon is a horizontal line drawn a distance below the crown equal to 6418.9/693 = 9. 26 ft. The moment at any point is equal to the thrust H_o multiplied by the vertical distance from such point to the equilibruim polygon.

Temperture

Expansion joints in the concrete were allowed every 50 ft. and consisted of a few sheets of tar paper inserted in the joint. When not reinforced concrete will, under such circumstances crack at intervals, its maximum deformation under stresses not being equal to its maximum temperature deformations. It is to be assumed that concrete when reinforced will not stretch more than plain concrete, as seems probable, then no amount of reinforcement can entirely prevent contraction cracks. The reinforcement can entirely, however, force such cracks to take place as they do in a beam,--at such frequent intervals that the requisite deformation takes place without any one crack becoming large. These temperature stresses obtained were very safe and are entirely taken up by the steel in the arch.

-LOADING-

Dead Load-

Concrete, including reinforcement, = 150# per cubic foot.

Asphalt pavement, including 6" concrete foundation, filling of gravel under pavement, also street car construction, complete is 140# per cubic foot.

Live Load-

Street car = 35 tons } Chicago Bridge Dept.
Sprinkler = 42 " } (See fig.A, plate 1.)

Uniform load of 100# per square foot over the rest of bridge roadway. Sidewalk load equals 60# per square foot.

-PANEL LOADS-

Live Load-

(See fig.B, plate 1.) Car and sprinkler are shown as regards lenths over diaphragm. We are considering them as placed side by side so as to obtain the greatest weight possible over diaphragm. (Note ! sketch does not show them side by side.)

We therefore, have over the diaphragm approx-imately one half of each, that is, 42/2+35/2+ =77/2 tons. This (77/2 tons) is concisdered as distributed evenly over the (33 ft.) width of the diaphragm.

77/2 x 2200# = 84700# = 84.7 kips.

Remaining roadway of 14 feet = 100# per sq. foot. 100# x 14x15 = 21 kips.

Sidewalk = 60#/ sq. ft. 60x2x10x15 = 18 kips.

Total = 84.7 + 21 + 18 = 123.7 kips.

This is evenly distributed over 33 foot diaphragm. (For calculations we consider diaphgram as 12" wide across bridge.

∴ Live panel load = 123.7/33 = 3.75 kips per ft. width of bridge.

-DEAD PANEL LOADS-

Note! (See fig.C, plate 1.) Dead panel load takes in material from lines AA to BB.

P_1 y = 36' (fig.D, plate 1.)

Assume diaphragm as 18" thick = 1 foot-6"

Its volume = 36' x 1 5/10x1' = 54cu.ft.

54x150# = 8.1kips.

Distance a_1 of arch rib is 17 feet.

Thickness = 4 1/2 feet. Width taken as 1 ft.

Volume = 17x4 1/2x1 = 77 cubic feet.

77x150# = 11.55 kips.

Roadway floor-slab assumed as 8" deep = 2/3 feet.

Weight per foot of length across the bridge is

15 x 1x2/3x150# = 1.5 kips

Sidewalk assumed as 6" deep.

(2 x 15 x 10 x 1/2 x 150#) ÷ 33 = .75k. per foot width of bridge.

8" pavement,; consisting of asphalt etc. Weight per ft. width of bridge is 140x15x2/3 1.4 kips

Total dead load =
8.1 + 11.55 + 1.5 + .75 + 1.4 = 23.30 kips

Total Live load = 3.75 kips

Total = P_1 = 27.05kips

P_2

$y_2 = 26'$ 26x1 1/2x1 = 39cu.ft. $a_2 = 17'$

Thickness = 41/4 feet 39x150# = 5.85k

I7 x 4 I/4 x I x I50# = I0.95 kips

Floor slab = I.5 K. Pavement = I.4 K.

$-P_4-$

y_4 = I2'   I2 x I I/2 x I = I8 cu. ft.

a_4 = I6.5 x 3.5 x I x I50# = 8.7 K.

I8 x I50# = 2.7 K.

Sidewalk, floor, pavement., = 3.65 K.

D. L. = I5.05 K.

L. L. = <u>3.75 K.</u>

Total = I8.8 K. = panel load P .

$-P_5-$

y_5 = 8 feet.   8 x I I/2 x I x I50# = I.8 K.

a_5 = I5.5x3.25 ft.   8 x I I/2 x I x I50# = I.8 K.

I5.5 x3.25 x I x I50# = 7.65 K.

Sidewalk., etc., 3.65 K.   D. L. = I3.I K.

L. L. = <u>3.75 K</u>

Panel load P_5.   Total = I6.85 K.

$-P_6-$

y_6 = 6'   6xI I/2xIxI50 = I.35 K.

a_6 I5x3   I5 x 3 x I x I50# = 6.75 K.

Sidewalk, pavement, etc., = 3.65 K.

D. L. = II.I K.

L. L. = <u>3.75 K.</u>

Total = I5.5 kips panelload P_6 .

Total dead load equals

$$5.85 + I0.95 + I.5 + .75 + I.4 = 20.45$$

Live load equals 3.75

Total Panel load $P_2 = 3.75 + 20.45 = 24.20$ K.

– P_3 –

$y_3 = I8'$ $I8 \times I\ I/2 \times I = 27$ cu. ft.

$a_3 = I6.5$ ft.   thickness is 3.75 ft.

$I6.5 \times 3.75 \times I \times I50\# = 9.285$ k. $27 \times I50\# = 4.05$ k.

Floor, sidewalk, pavement same as above = 3.65 K.

D. L. = I6.985 K.

L. L. = 3.750 K

Total = 20.74 K. panel load P_3

– P_7 –

$y_7 = 5$ft. $5 \times I\ I/2 \times I = 7.5$ cu. ft.

$a_7 = I4.5 \times 3$ $I4.5 \times 3 \times I \times I50\# = 6.525$ K

$7.5 \times I50\# = I.I25$ K.

Floor, sidewalk, and pavement = 3.65 K.

D. L. = II.30 K.

L. L. = 3.75 K

Total = I5.05 K. panel load P_7

DESIGN OF SIDEWALK AND FLOOR SYSTEM.

The floor system consists of reinforced concrete slabs resting on floor longitudinal girders or stringers, spaced as shown. The two inside stringers are spaced 10'-0" c.to c., being directly under the center lines of the tracks. The sidewalk slabs are supported by the outside stringers and by beams caeeiwd on the ends of cantilevers placed every 15' (Fig.B. Plate-11.)

-THE SIDEWALK-

Live load on sidewalk = 60# per square foot. The width of the sidewalk, and hence the span of the the slab, is taken as 10'-0". From Turneaure and Maurer Reinforced Concrete Construction. Table 21,(7) page 298, we find that for this span and loading a 6" slab may be used, for a value of the bending moment of 1/12 wl . The required area of steel per foot of width for this slab is .385 sq. in. This will be furnished by steel rods.

LONGITUDINAL BEAM AT OUTSIDE OF WALK.

A rectangular cross-section will be used.

Span of beam = 15'-0".

Dead load:

Assume weight of beam = 300# per linealfoot, and weight of hand rail = 650#/ft.

Weight of sidewalk slab per sq. ft. = 73#.

Weight of slab taken by beam (1/2 sidewalk)-	5x15x73	= 5480#
Weight of beam =	15x300	= 4500
Weight of rail =	15x650	= 9750
Live load at 60#/ ft. =	60x5x15	= 4500
	Total	24230#

Max. bending moment = 1/8wl.

= 1/8x24200x15x12 = 453000 in. lbs.

$M_s = f_s$ Ax 7/8d. $M_c = f_c$ x1/6bd.

Assume "b" = 12"

$$d^2 = \frac{6MC}{12x600} = \frac{540000x6}{12x600} = 450 \therefore d = 21.2$$

use 22".

$$A = \frac{8}{7} \cdot \frac{Ms}{f_s d} = \frac{8x540000}{7x22x16000} = 1.76 \text{ sq. in.}$$

This area is furnished by 5/8" bars spaced 2 c. to c. Total depth of beam should be made 25".

-DESIGN OF CANTILEVER-

This will be designed as a cantilever beam having a single load at the end, due to the weight oft he beam just designed.

Weight of one girder, including slab and rail 24000#

The general dimension of the beam will be assumed, and the reinforcing figured ; as it will be of such shape as to be of nearly uniform strength, the uniform load due to its own weight will not be considered.

Maximum moment due to load P at end M Pl.

M = 24000x15 = 360000 ft. lbs, = 4320000 in. lbs.

Shear at any point = 24000#. Therefore the required area at an allowablw shearing stress of 100#/sq.in. = 240 sq.in. Make the beam 12x20 at least.

$$A = \frac{8}{7} \frac{M_s}{f_s d}$$

-FLOOR SLAB-

The live load upon the floor slab is 100#/sq.ft. as per specifications. From Table 21, Turneaure and Maurer, as for the sidewalk slab, the thickness

of slab for a span of 11.5 ft. and a loading of 200#/ft. (M 1/12 wl^2) is 8". .547 sq. inches of steel are required ; this is furnished by 3/8" bars, 2" apart.

-GIRDER-

This will be designed as a T beam with a flange thickness of 8" (floor slab.)

Assume weight of girder	= 1000#	per	lineal	ft.
Weight of slab	98	"	"	"
Live load	100#	"	"	"
(consider as dead load)	198#			

Dead load on girder

Slab at 198#/sq. ft.	1980#
Girder	1000
Total-	2980#

Bending moment $M_D = 1/8 \times 2980 \times \overline{15}^2 \times 12 = 1008000$ "#

The live load is furnished by a sprinkler weighing 42 Tons per car. This is on two trucks 16'4" c. to c. 21 Tons per truck. The maximum moment occurs with the load at the centre.

$M_L = 42000 \times 15/4 \times 12 = 1890000$ "#

Total moment $= M_D + M_L = 1890000 + 1008000$

$= 2898000$ "#

-SHEAR-

The maximum shear occurs at the supports when the truck is just leaving the span.

This shear is 21 tons = 42000#

Dead load shear = 2980x15/2=21400
63400# Total shear

Allowable shearing stress = 100#/sq.in.

Area of concrete required = 63 4 sq. in.

Owing to the arched form of the spandrels, this area will be provided for at the ends. It will never be required at the center, as the sketch Fig. "C", Pl. 11. shows. The maximum shear which can exist at the center of the span is 42000/2=21000#, and it may be considered as varying uniformly towards the end. This shear would require only 210 sq. in, We will use 560 □", and assume that the arch will take the shear between the quarter point and the supports.

560 sq. in.=14"x40", 16"x35", or 20"x"8" section Try 16" x 35", with 8" flange. From plate X, Page 284, Tureaure and Maurer, for t/d= 8/35=.228,

and f_c=600, j=.905 and M/bd^2=85

jd= .905x 35=31.7

bd^2= 2898000/85=34100 .: b=34100/35 = 37.9" say 28"

$$A = \frac{M}{f\ jd} = \frac{2898000}{31.7\times16000} = 5.71\ \square''$$

Use 6 at 7/8" 3.60 and 5 at 3/8 2.20, Total area 5.80 □" See figure "D" Pl. II for arrangement of rods. Distance of centre of gravity of reinforcement from ℄ of bottom row=2.2x3.5/5.80=1.53

Total depth of girder 35 + 1.[illegible] + 2.5=39".

These rods will be turned up at intervals, to assist in over coming the shear. The points at which they may be turned up are found from the following formula:

$$X = 1/A\sqrt{a_1 + a_2 + a_3 + \cdots + a_n}$$

X being the unbent lengths of rod required to resist bendind moment and a_1, a_2, etc. the areas of the rods.

-LENGTHS-

No. of rod.	$a_1 + a_2 +++ a_n$	X
1	.44	4.13 ft.
2	.88	5.85 "
3	1.32	7.15 "
4	1.76	8.26 "
5	2.20	9.25 "
6	2.80	10.42 "
7	3!40	11.50 "
8	4.00	12.45 "
9	4.60	13.35 "
10	5.20	14.20 "
11	5.80	15.00 "

The length of rod required to develope a bond strength equal to the working strength equals 16000/4x75 = 53.4d. This 47" for a 7/8" rod; and is amply provided for, as shown by the above table. The rods will be arranged as follows, the unbent length being given in each case:

2 at 6'-0"
2 " 9'-0"
2 " 11'-0"
2 " 13.0" } These rods to continue over supports.
3 " 15'-0"

We will now find the point at which the shearing stress becomes equal to 100#/sq.in. as beyond that point stirrups will be required. The stress at the centre equals 43000/ 560=76#/per sq.in.

The maximum shear at any point distant x from the centre= 3X/ 15x9240+3X/15x43000+43000, and it becomes equal to 100#/sq.in. when [illegible]000-[illegible]000= 3X/15x [illegible].

[illegible]=[illegible]40[illegible] ∴ x=1.[illegible]

Stress carried by concrete equals 30#/sq.in.

" " " steel " [illegible] " "

Use 1/3" [illegible] [illegible] [illegible] loop. [illegible] [illegible] [illegible]/ sq.in.= [illegible] = [illegible] [illegible] = [illegible] = [illegible]

Space 9" [illegible]

-INVESTIGATION OF FLANGE FOR SHEAR-

The width of the web of the girder is less than the width of tread [illegible] of rails so that the load [illegible] under the wheel is the condition of a load with a load P at the point where the wall lies. [illegible] will be considered [illegible] [illegible] [illegible] a cantilever with a load P at the wall.

The [illegible] equals (53.5–[illegible])½ [illegible] = 3[illegible].35'. The weight of the [illegible] is [illegible] = [illegible].

[illegible]

[illegible]. This is the required width of [illegible]

[illegible]

$$A = \frac{[illegible]}{s} = [illegible]$$

[illegible]

-ABUTMENT-

(See Plate 4.)

Figure "A".

In figuring size of abutment it is necessary to know the maximum thrust at springing line due to arch action and also the vertical force acting downwards of that amount of concrete which we will consider as consisting the abutment.

Column marked "C" and block of solid concrete marked "B" in the sketch are those portions which shall constitute the abutment. (Column C is hollow as shown in both figs. A and B, its interior being filled completely with earth A.)

-VERTICAL FORCE-

Weight of column marked "C".

Eearth: 44'x9'x37'x120#=1758240#.

Concrete: (2x3'x12'x44'x150#)
(2x3'x37'x44'x150#) = 1549800#

Total weight=1758240#+1549800#= 3308000#.

Weight per foot width of bridge:

3308000÷33=100000#=100 kips'

Weight of block "B":

30'x32'xI'xI50# = I44000#per ft. width of bridge.

Total downward verticsl pressure:

I44000# + I00000# = 244 kips

Center of gravity of the two portions, column "C"and block "B" is found and from it is drawn verticallydownward a line. Maximum axial thrust of arch (2I7 k.) is drawn in direction of action. This line cuts the vertical line at point as shown in Fig. A. From this point,and to a certain scale is laid off 244 K. & 2I7K.

Their reaultant as shown must and does pass within the middle third of the block "B". Therefore the dimension of "B" and "C" are correct.

-PILES-

Vertical downward force of abutment as found above : 3,308,000# (See Fig."C" Plate 4)

Formula for resistance "R" of one pile:

$R = \frac{2wh}{s+I}$ h = 20 ' = drop of hammer. W = 3000# = weight of hammer. s = I" = distance pile is imbedded at last blow of hammer.

$R = \frac{2 \times 3000 \times 20}{I+I} = 60000\#$ $\frac{3308000}{60000} = 56$ piles necessary.

PLATE I.

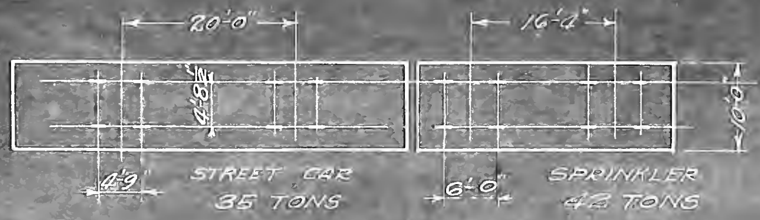


FIGURE "A"

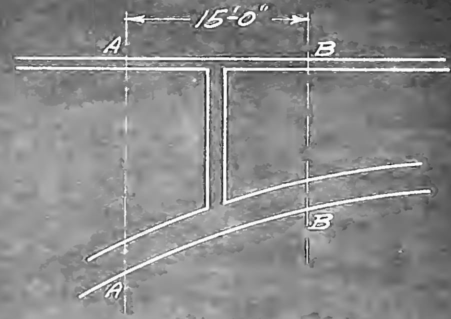


FIGURE "C"

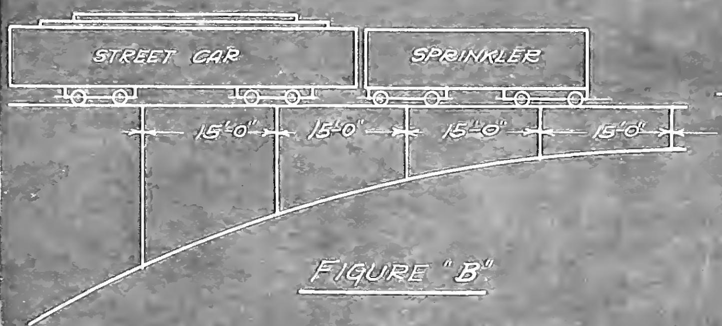


FIGURE "B"

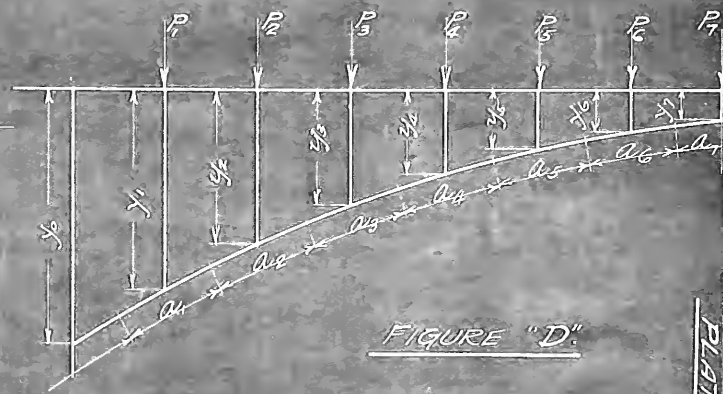


FIGURE "D"

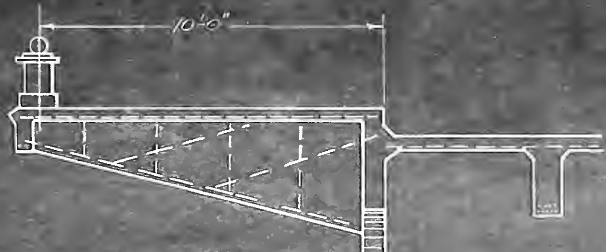


FIGURE "A"
SIDEWALK BRACKET.

FIGURE "B".
SIDE WALK.

FIGURE "C".

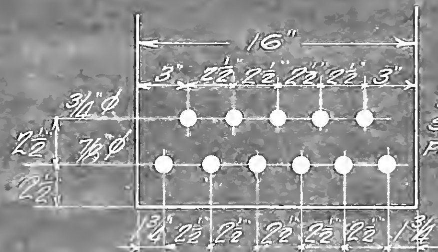


FIGURE "D".
STEEL IN FLOOR GIRDER.

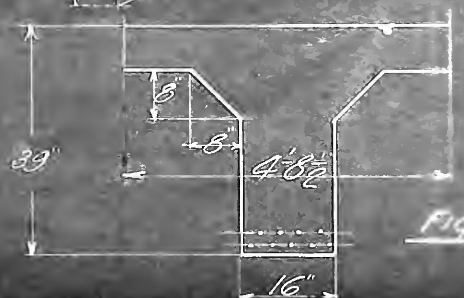


FIGURE "E".

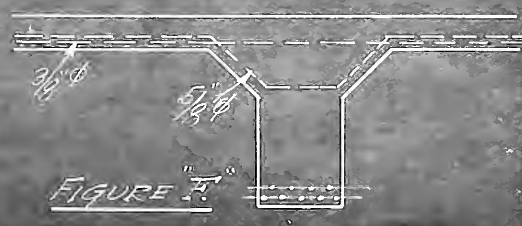


FIGURE "F".

PLATE 3.


PRELIMINARY DIVISIONS
FINAL DIVISIONS
Rad.=147.2'
44'-0"
210'-0"
A
B
a b c d
PRELIMINARY DIVISIONS
FINAL DIVISIONS.
LENGTH ARC AB = Rθ = 147.2 θ = 116.97 FEET.
$\frac{\Sigma i}{N} = \frac{3.4769}{14} = .2484$
$ds \times i = 2.074$
SEE TABLE A

PLATE 4.

FIG. A.

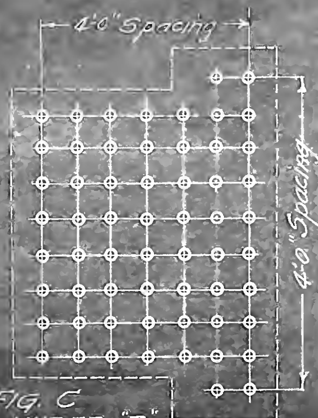
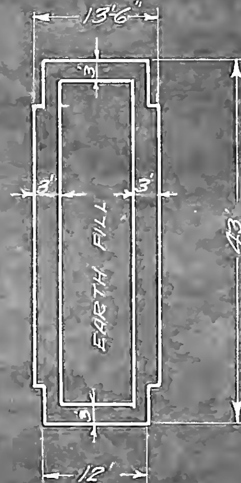


FIG. C
PILING UNDER "B"



SECTION MN.

FIG. B

TABLE "A".

PROPERTIES OF PRELIMINARY EQUAL DIVISIONS						PROPERTIES OF FINAL DIVISIONS.			
No. OF DIVIS.	DEPTH D	I_c	$15 I_s$	$I = I_c + 15 I_s$	$i = \frac{1}{I}$	i	ds	I	d
1	3.00	2.251	.253	2.504	.3993	.408	4.162	2.45	3.01
2	3.01	2.272	.258	2.530	.3952	.4032	4.200	2.48	3.02
3	3.09	2.459	.269	2.728	.3667	.398	4.526	2.515	3.04
4	3.18	2.676	.285	2.961	.3376	.390	4.821	2.562	3.06
5	3.23	2.809	.295	3.104	.3222	.3801	5.051	2.631	3.09
6	3.28	2.944	.304	3.248	.3078	.3680	5.397	2.718	3.12
7	3.46	3.451	.338	3.789	.2644	.351	6.278	2.848	3.15
8	3.66	4.079	.378	4.457	.2225	.328	7.382	3.05	3.20
9	3.78	4.508	.403	4.911	.2035	.298	8.200	3.36	3.38
10	3.98	5.258	.445	5.703	.1753	.261	9.474	3.84	3.50
11	4.22	6.261	.495	6.756	.1480	.218	11.252	4.58	3.64
12	4.41	7.139	.544	7.683	.1309	.177	12.732	5.65	3.90
13	4.68	8.538	.616	9.154	.1093	.135	15.25	7.410	4.28
14	4.96	10.172	.695	10.867	.0932	.0981	18.050	10.200	4.76
					3.4769		116.974		

.5 % STEEL ABOVE & BELOW AXIS. LENGTH ARC = $R\theta = \frac{45.58°}{57.29°} \times 147.2 = 116.97$.

AVERAGE $i = \frac{\Sigma i}{n} = \frac{3.4769}{14} = .2484$

$ds \cdot i = 116.97/14 \times .2484 = 2.074$.

(SEE PLATE 3).

Point	x	y	x^2	y^2	m_L	m_R	$[m_L+m_R]y$	$[m_R-m_L]x$
1	2.281	.0011	4.331	.0000	− 44.52	− 44.52	− .069	0.0
2	8.500	0.202	39.199	.0408	− 96.01	− 96.01	− 38.784	0.
3	13.200	.411	112.870	.169	− 165.00	− 165.00	− 131.520	0.
4	14.50	.906	235.56	.821	− 243.3	− 243.3	− 440.86	0.
5	20.584	1.521	415.52	2.3104	− 396.2	− 396.2	− 1204.446	0.
6	25.840	2.340	681.50	5.4756	− 560.7	− 560.7	− 2624.076	0.
7	31.590	3.502	991.96	12.250	− 764.2	− 764.2	− 5349.400	0.
8	38.240	5.100	1466.49	26.0100	− 1082.2	− 1082.2	− 11038.440	0.
9	45.616	7.262	2126.68	52.707	− 1446.4	− 1446.4	− 21088.796	0.
10	54.150	10.260	3019.48	105.27	− 1999.3	− 1999.3	− 41025.640	0.
11	63.6	14.310	4266.18	204.49	− 2699.8	− 2699.8	− 77274.0	0.
12	72.92	19.72	5976.59	388.89	− 3600.5	− 3600.5	−141984.0	0.
13	84.9	26.396	8335.48	696.74	− 4908.5	− 4908.5	−259168.800	0.
14	98.34	37.680	11653.0	1418.680	− 6538.6	− 6538.6	−493002.9	0.
Σ		129.611	39324.83	2913.858	− 49070.46		1054471.733	0.0

CASE I

SPAN COVERED ENTIRELY WITH LIVE LOAD

$H_0 = 174.5$ KIPS

$V_0 = 0$

$M_0 = 132$ KIP-FEET

TABLE "B"

TABLE "C"

POINT	THRUST		ECCENTRIC DIST		BENDING MOMENT	
	LEFT	RIGHT	LEFT	RIGHT	LEFT	RIGHT
1	174.8	174.8	+ .558	+ .558	+ 87.65	+ 87.65
2	174.8	174.8	+ .407	+ .407	+ 71.21	+ 71.21
3	174.8	174.8	+ .209	+ .209	+ 36.68	+ 36.68
4	176.1	176.1	+ .207	+ .267	+ 74.0	+ 74.0
5	176.1	176.1	+ .074	+ .074	+ 1.3	+ 1.3
6	176.1	176.1	− .1165	− .1165	− 30.5	− 20.5
7	179.0	179.0	− .113	− .113	− 20.2	− 30.2
8	179.0	179.0	− .335	− .335	− 60.0	− 60.0
9	183.8	183.8	− .252	− .252	− 46.4	− 46.4
10	183.8	183.8	− .373	− .373	− 70.3	− 70.3
11	191.4	191.4	− .350	− .35	− 67.8	− 67.8
12	191.4	191.4	− .149	− .149	− 28.5	− 28.5
13	202.8	202.8	− .817	− .817	− 165.5	− 165.5
14	217.7	217.7	+ .788	+ .788	+ 171.4	+ 171.4
	ABOVE	GRAPHICALLY	SHOWN	ON	PLATE 5	

CASE I

SPAN COVERED ENTIRELY WITH LIVE LOAD

POINT	e/h	M/bh^2f_c	f_c	$1/k$	f_s
1	.185	.107	597	.98	2210
2	.134	.089	598	.84	1851
3	.0686	.057	532	.504	1632
4	.0872	.068	513	.610	1584
5	.0242	.0234	403	.240	2960
6	.0374	.0342	427	.320	4840
7	.0359	.0339	418	.318	5020
8	.1048	.0773	522	.700	3716
9	.0746	.0625	464	.560	4050
10	.1065	.0778	508	.710	4925
11	.0974	.0741	410	.662	4120
12	.0382	.0354	372	.320	4238
13	.1908	.1089	575	1.001	2184
14	.1644	.1012	519	.941	3850
	FROM TURNER & MAURER [illegible]				

CASE I

$$f_s = nf_c\left[1 - \frac{d}{kh}\right]$$

TABLE-C-2


27.05
24.2
20.75
18.8
16.85
15.5
15.05
15.5
16.85
18.8
20.75
24.2
27.05
1 2 3 4 5 6 7 8 9 10 11 12 13 14
P_1 P_2 P_3 P_4 P_5 P_6 P_7 P_8 P_9 P_{10} P_{11} P_{12} P_{13}
H_o
CASE I
LIVE LOAD OVER ENTIRE SPAN
H_o = 174.5 KIPS
V_o = 0.0
M_o = 132 KIP-FEET

PLATE 5

PT	x	y	x^2	y^2	m_L	m_R	$[m_L+m_R]y$	$[m_R-m_L]x$
1	2.281	.001	4.331	.0000	− 34.52	− 34.52	− .069	0.0
2	8.5	.201	39.199	.0408	− 96.010	− 96.010	− 38.784	0.
3	13.2	.411	112.87	.169	− 165.00	− 165.00	− 131.52	0.
4	14.5	.906	235.56	.821	− 243.3	− 241.45	− 440.123	+ 26.82
5	20.584	1.521	415.52	2.310	− 396.2	− 375.4	− 1172.832	+ 220.104
6	25.84	2.34	681.50	5.4756	− 560.7	− 519.45	− 2527.551	+ 1076.625
7	31.59	3.502	991.96	12.25	− 764.2	− 646.2	− 4932.90	+ 3728.5
8	38.24	5.1	1466.49	26.01	− 1082.2	− 966.0	− 10445.82	+ 4450.46
9	45.616	7.262	2126.68	52.707	− 1397.3	− 1247.25	− 19199.433	+ 6910.0
10	54.15	10.26	3019.48	105.27	− 1999.3	− 1725.0	− 38211.318	+ 15072.0
11	63.60	14.31	4266.18	204.49	− 2699.8	− 2294.7	− 71485.605	+ 25764.36
12	72.92	19.72	5976.59	308.89	− 3600.5	− 3052.5	− 131197.16	+ 39960.16
13	84.90	26.396	8335.48	696.74	− 4908.5	− 4139.6	− 237867.2	+ 65380.0
14	98.34	37.68	11653.0	1418.68	− 6538.6	− 5502.0	− 453704.88	+ 101062.0
Σ		129.611	39324.83	2913.858	− 24486.13 − 43490.21	− 19004.08	− 971355.195	+ 264229.029

CASE II

LIVE LOAD ON LEFT HALF ONLY

H_o = 165.5 KIPS

V_o = 3.36 ″

M_o = 207 KIP-FEET

TABLE "D"

TABLE "E"

POINT	THRUST		ECCENTRIC DIST		BENDING MOMENT	
	LEFT	RIGHT	LEFT	RIGHT	LEFT	RIGHT
1	165.6	165.4	− .0362	− .1988	− 5.99	− 21.3
2	165.6	165.4	− .279	− .426	− 46.43	− 70.5
3	165.6	165.4	− .1928	− .438	− 31.9	− 72.4
4	166.7	166.6	− .1437	− .482	− 23.9	− 80.2
5	163.7	166.6	− .328	− .434	− 54.7	− 72.3
6	166.7	166.6	− .379	− .628	− 65.7	− 88.1
7	169.5	169.0	− .338	− .476	− 57.3	− 80.6
8	169.5	169.0	− .579	− .637	− 80.0	− 90.8
9	174.8	172.9	+ .1218	− .625	− 21.3	− 108.1
10	174.8	172.9	− .557	− .621	− 97.6	− 107.3
11	182.6	178.1	− .521	− .663	− 95.1	− 118.8
12	182.6	178.1	− .382	− .0663	− 69.8	− 11.8
13	194.0	186.8	− .915	− .176	− 181.0	− 39.2
14	209.4	198.8	+ .176	+ 1.100	+ 36.9	+ 219.0
	ABOVE GRAPHICALLY SHOWN ON PLATE 6					

CASE II

LIVE LOAD ON LEFT HALF ONLY

POINT	e/h	M/bh^2f_c	f_c	$1/k$	f_s
1	.0121	.0151	303	.120	4110
2	.0924	.0718	463	.642	3680
3	.0632	.0557	427	.431	3690
4	.0470	.0426	416	.391	4112
5	.162	.0995	401	.922	2810
6	.1294	.088	539	.818	2998
7	.1072	.0778	514	.710	3260
8	.1623	.0996	600	.930	2740
9	.0361	.0348	372	.311	4290
10	.158	.0981	561	.914	2980
11	.143	.0927	537	.862	3074
12	.098	.0741	432	.780	3262
13	.213	.1156	592	1.210	1840
14	.037	.0351	327	.310	3772
	FROM TURN. & M. PLATE XIIC-XIV				

CASE II [LEFT]

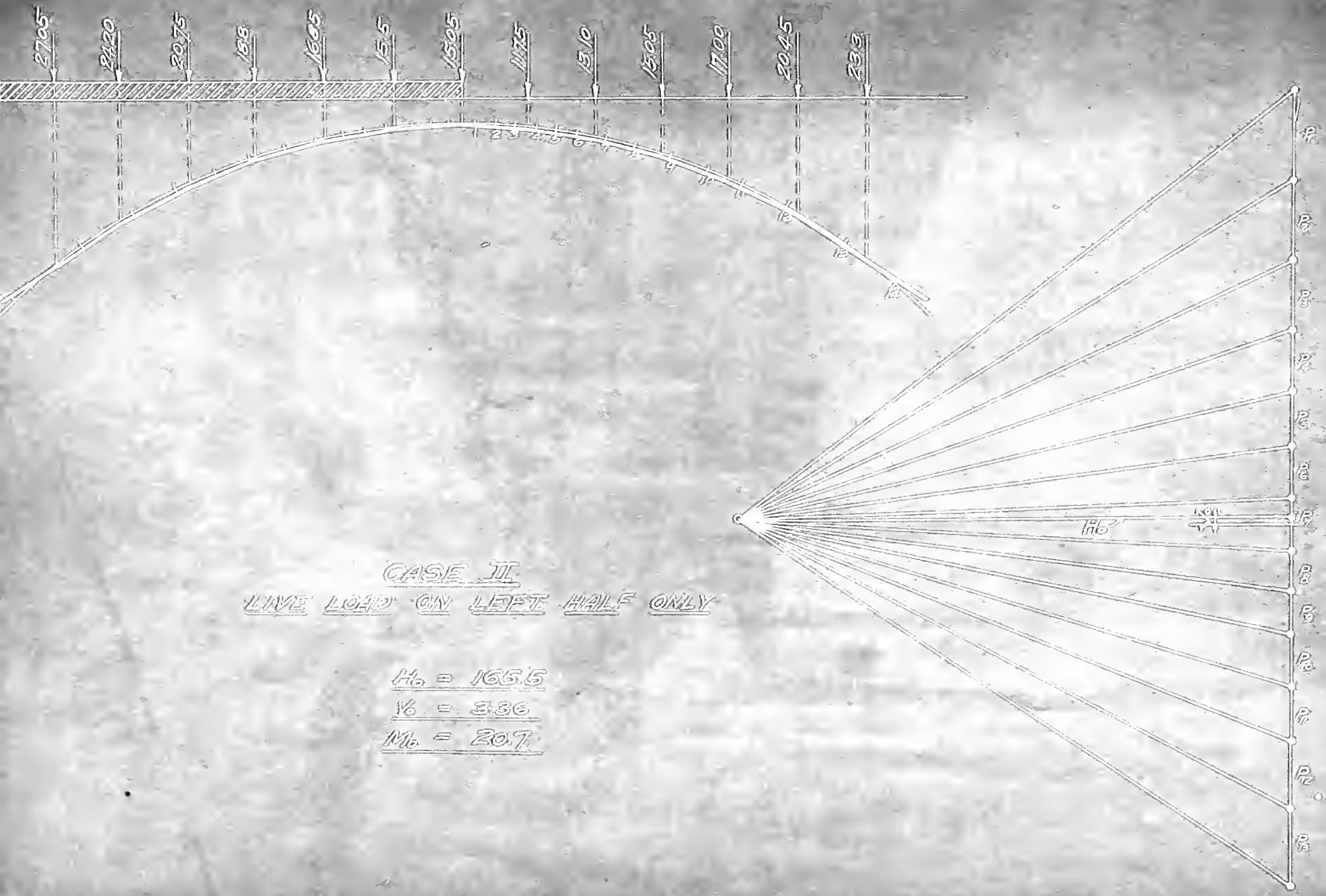
$$f_s = nf_c\left[1 - \frac{d}{kh}\right]$$

TABLE E-3

POINT	e/h	M/bh^2f_c	f_c	$1/k$	f_s
1	.0428	.0381	428	.362	4700
2	.1410	.0921	582	.860	3124
3	.144	.093	579	.872	3086
4	.158	.0992	597	.895	2960
5	.141	.093	564	.860	3240
6	.170	.103	599	.950	2594
7	.151	.0957	588	.890	2896
8	.168	.102	567	.948	2640
9	.185	.108	602	.981	2408
10	.178	.106	571	.970	2562
11	.182	.106	509	.981	2432
12	.017	.018	295	.160	3915
13	.0428	.0385	335	.358	2982
14	.232	.1197	559	1.210	1430
	FROM TURN. & MAUR. PLATES XIII & XIV				

CASE II [RIGHT]

$$f_s = nf_c[1 - d/kh]$$


27.05
24.20
20.75
18.8
16.65
15.5
15.05
11.75
13.10
15.05
17.00
20.45
23.3
CASE II
LIVE LOAD ON LEFT HALF ONLY
Ho = 166.5
Vo = 3.36
Mo = 20.7

PT.	x	y	x^2	y^2	m_L	m_R	$[m_L+m_R]y$	$[m_R-m_L]x$
1	2.281	.001	4.331	.0000	- 44.52	- 44.52	- .069	0.0
2	8.5	.201	39.199	.0408	- 96.01	- 96.01	- 38.784	0.0
3	13.2	.411	112.87	.169	- 165.0	- 165.0	- 131.52	0.
4	14.5	.906	235.56	.821	- 243.3	- 243.3	- 140.86	0.
5	20.584	1.521	415.52	2.310	- 396.2	- 396.2	- 1204.448	0.
6	25.84	2.34	661.50	5.4756	- 560.7	- 560.7	- 2624.076	0.
7	31.59	3.502	991.96	12.25	- 764.2	- 764.2	- 5349.4	0.
8	38.24	5.10	1466.49	26.01	- 1082.2	- 1082.2	- 11038.44	0.
9	45.601	7.264	2126.68	52.707	- 1443.6	- 1443.6	- 20961.072	0.
10	54.15	10.26	3019.48	105.27	- 1963.8	- 1963.8	- 40299.176	0.
11	63.60	14.31	4266.18	204.490	- 2615.1	- 2615.1	- 74844.162	0.
12	72.92	19.72	5976.59	308.69	- 3436.0	- 3436.0	- 135515.84	0.
13	84.90	26.396	8335.48	696.74	- 4610.7	- 4610.7	- 243444.96	0.
14	98.34	37.68	11653.0	1418.68	- 6066.9	- 6066.9	- 457201.90	0.
Σ		129.611	39324.83	2913.658	-46956.46		-973092.707	0.

CASE III

LINE LOAD ON MIDDLE THIRD

H_o = 1567 KIPS

V_o = 0

M_o = 226.4 KIP-FEET.

TABLE "F"

TABLE "G"

POINT	THRUST		ECCENTRIC DIST		BENDING MOMENT	
	LEFT	RIGHT	LEFT	LEFT	LEFT	RIGHT
1	157.0	157.0	+ .572	+ .572	+ 90.10	+ 90.10
2	157.0	157.0	+ .483	+ .483	+ 76.204	+ 76.204
3	157.0	157.0	+ .426	+ .426	+ 67.8	+ 67.8
4	158.7	158.7	+ .6208	+ .6208	+ 98.7	+ 98.7
5	158.7	158.7	+ .456	+ .456	+ 72.384	+ 72.384
6	158.7	158.7	+ .204	+ .204	+ 32.38	+ 32.38
7	161.8	161.8	+ .0661	+ .0661	+ 10.65	+ 10.65
8	161.8	161.8	− .35	− .35	− 56.63	− 56.63
9	166.4	166.4	− .478	− .478	− 79.56	− 79.56
10	166.4	166.4	− .6749	− .6749	− 109.7	− 109.7
11	172.9	172.9	− .74	− .74	− 127.9	− 126
12	172.9	172.9	− .708	− .708	− 122.6	− 122.6
13	182.3	182.3	− .955	− .955	− 178.1	− 178.1
14	194.9	194.9	+ .348	+ .348	+ 67.09	+ 67.09
	ABOVE GRAPHICALLY SHOWN ON PLATE 7					

CASE III

LIVE LOAD ON MIDDLE THIRD

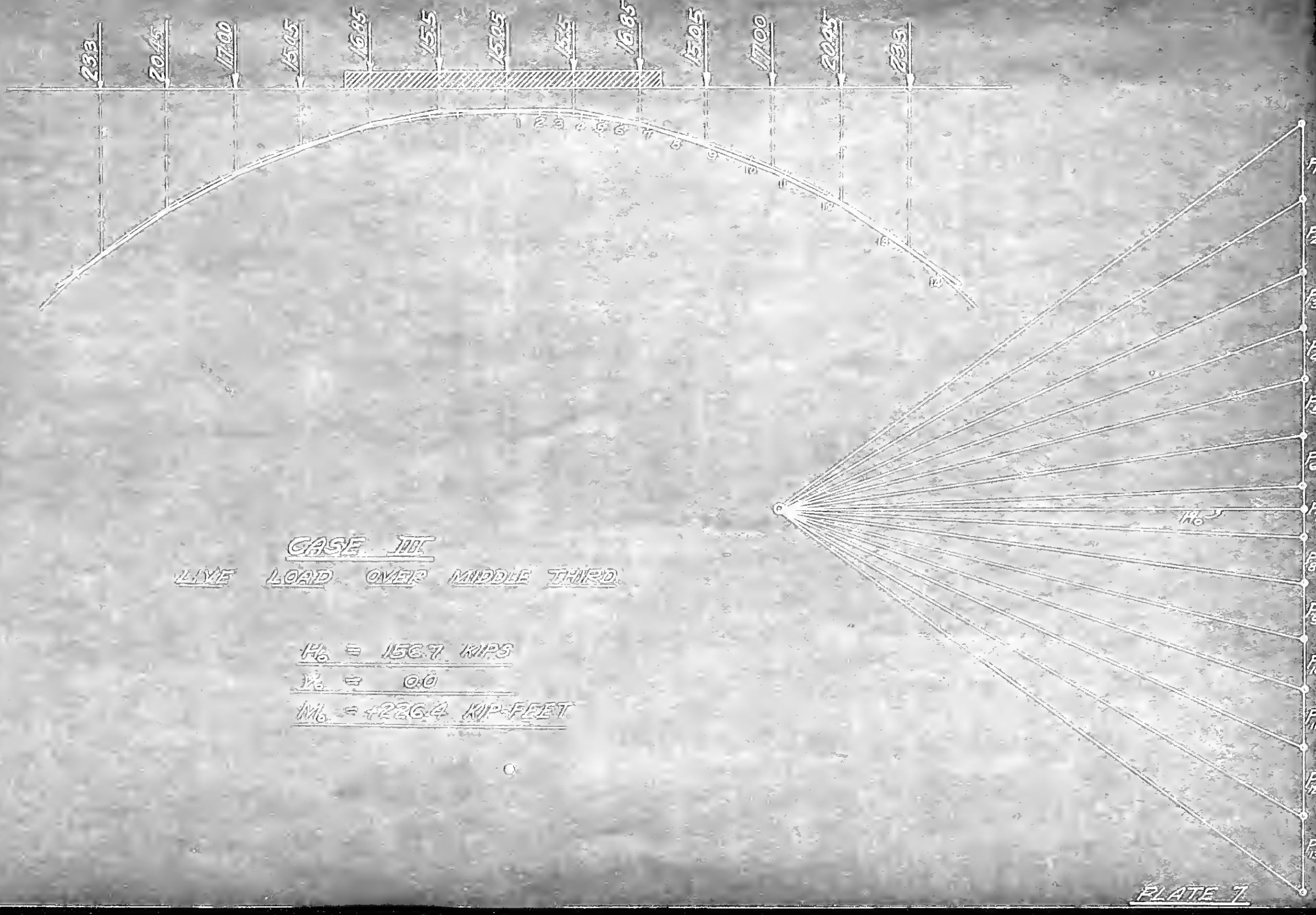
TABLE-G2

POINT	$\frac{e}{h}$	$\frac{M}{bh^2 f_c}$	f_c	$1/K$	f_s
1	.941	.119	598	1.020	2230
2	.1634	.107	581	.930	2905
3	.140	.092	550	.860	2800
4	.203	.115	603	1.040	3004
5	.150	.096	557	.890	2785
6	.065	.056	412	.490	4120
7	.020	.022	339	.210	4400
8	.110	.078	492	.740	3250
9	.140	.092	525	.860	2953
10	.182	.105	586	.980	2510
11	.198	.112	608	1.101	2250
12	.181	.105	533	.981	1804
13	.227	.120	563	1.10	1589
14	.071	.059	350	.551	3150
	FROM TURN. & MAUR. PLATE XII-XIV				

CASE III

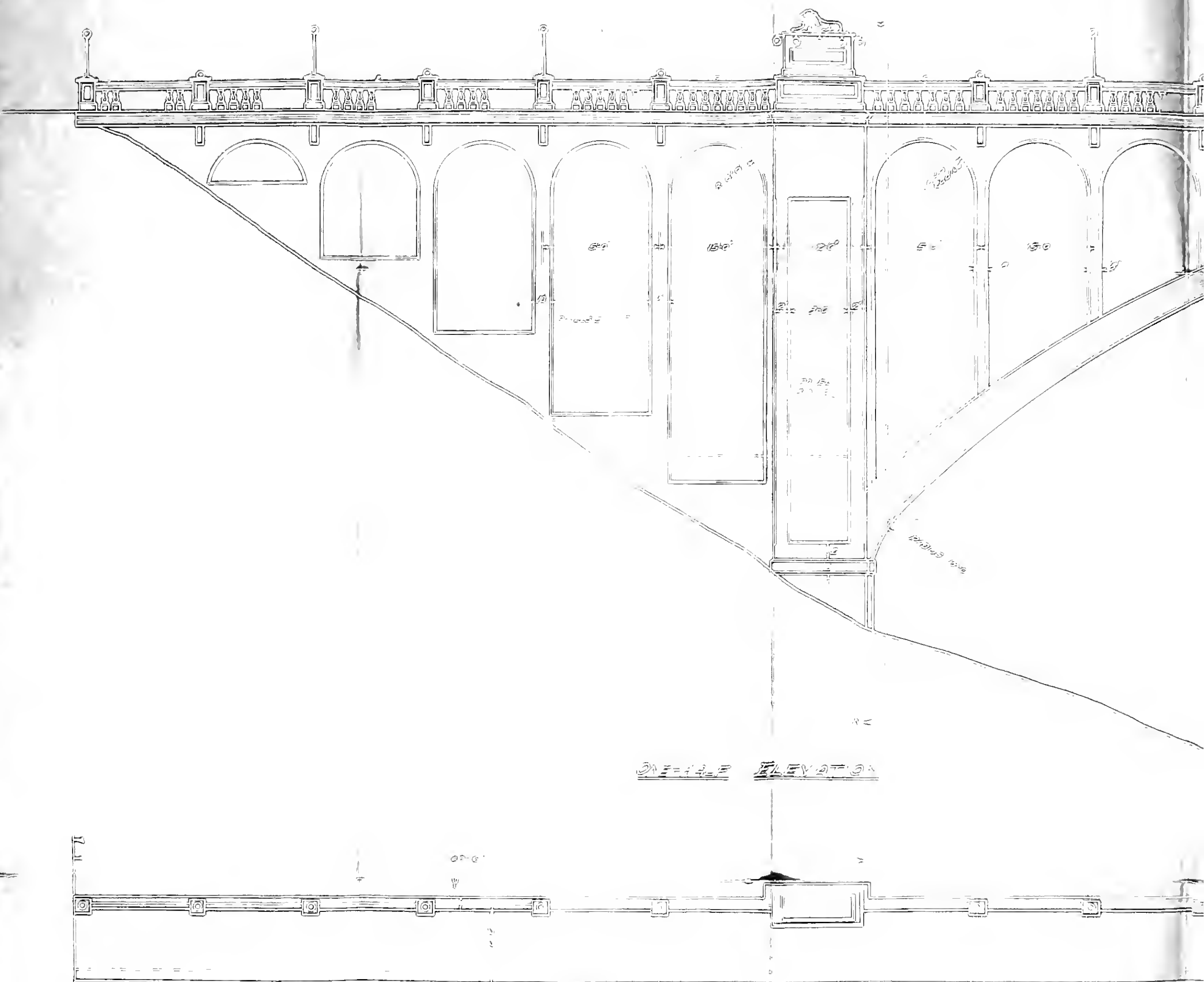
$f_s = nf_c\,[1 - d/kh]$

f_c from $M/bh^2 f_c$

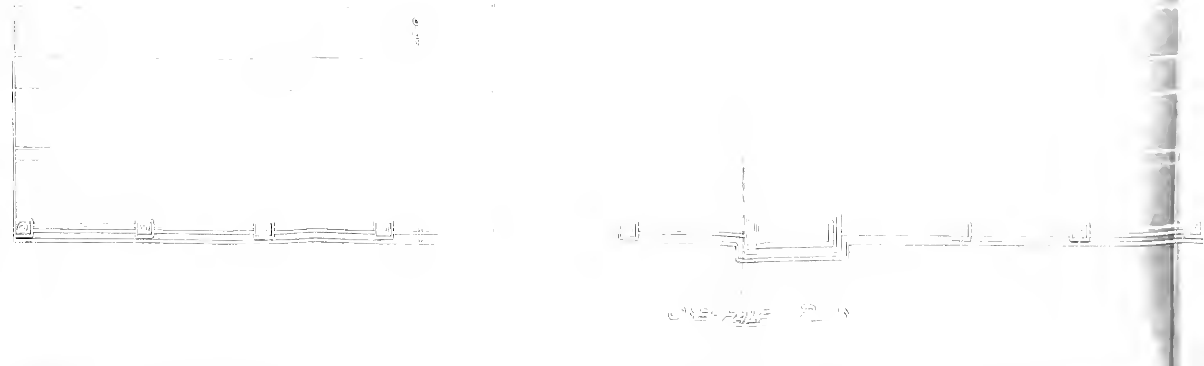


23.3
20.45
17.00
15.05
16.85
15.5
15.05
15.5
16.85
15.05
17.00
20.45
23.3
CASE III
LIVE LOAD OVER MIDDLE THIRD
H₀ = 156.7 KIPS
V₀ = 0.0
M₀ = +226.4 KIP-FEET
P1
P2
P3
P4
P5
P6
P7
P8
P9
P10
P11
P12
P13
H₀

PLATE 7



ONE-HALF ELEVATION



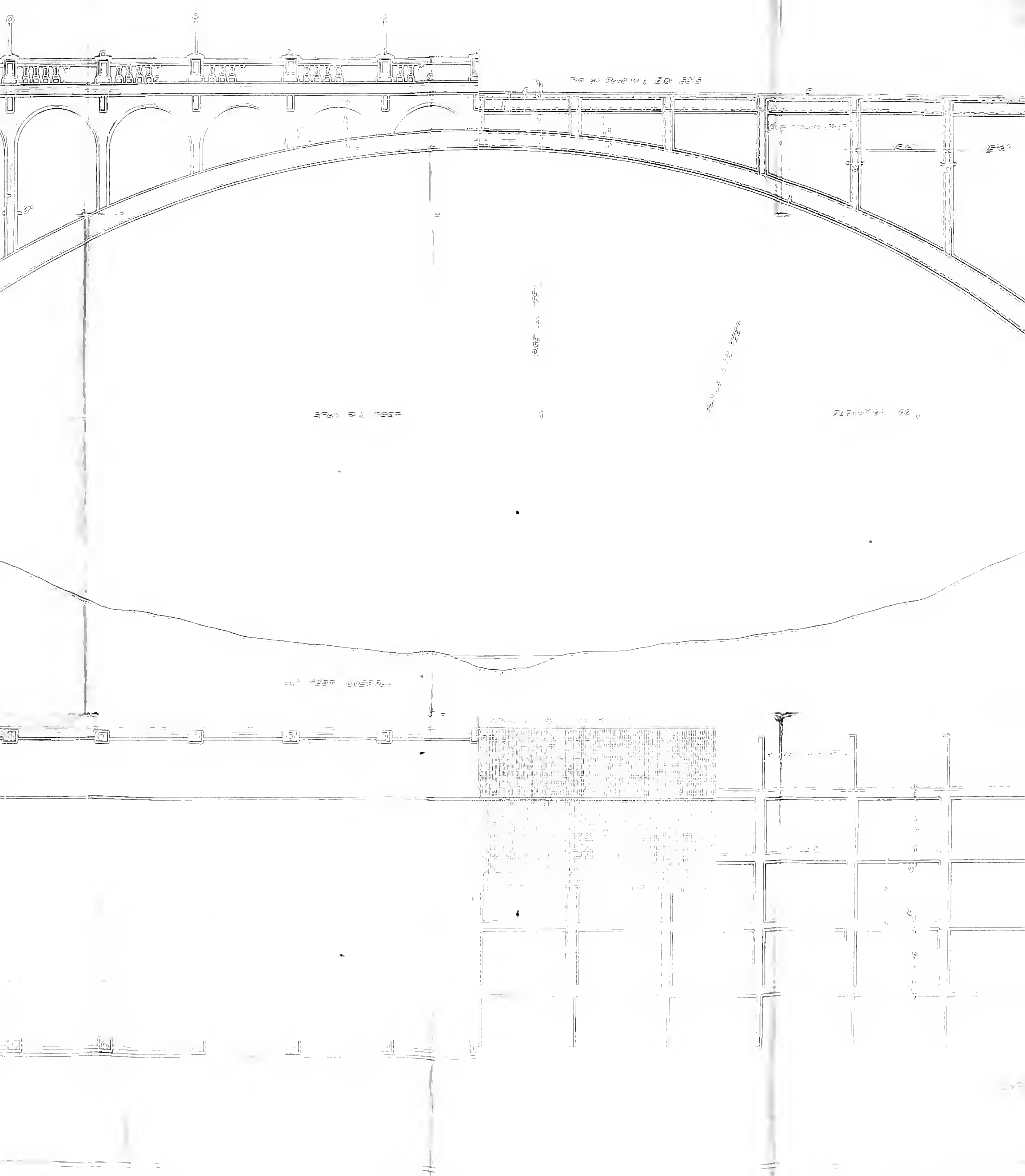
ONE-HALF PLAN

ARMOUR INSTITUTE OF TECHNOLOGY
CIVIL ENGINEERING DEPARTMENT

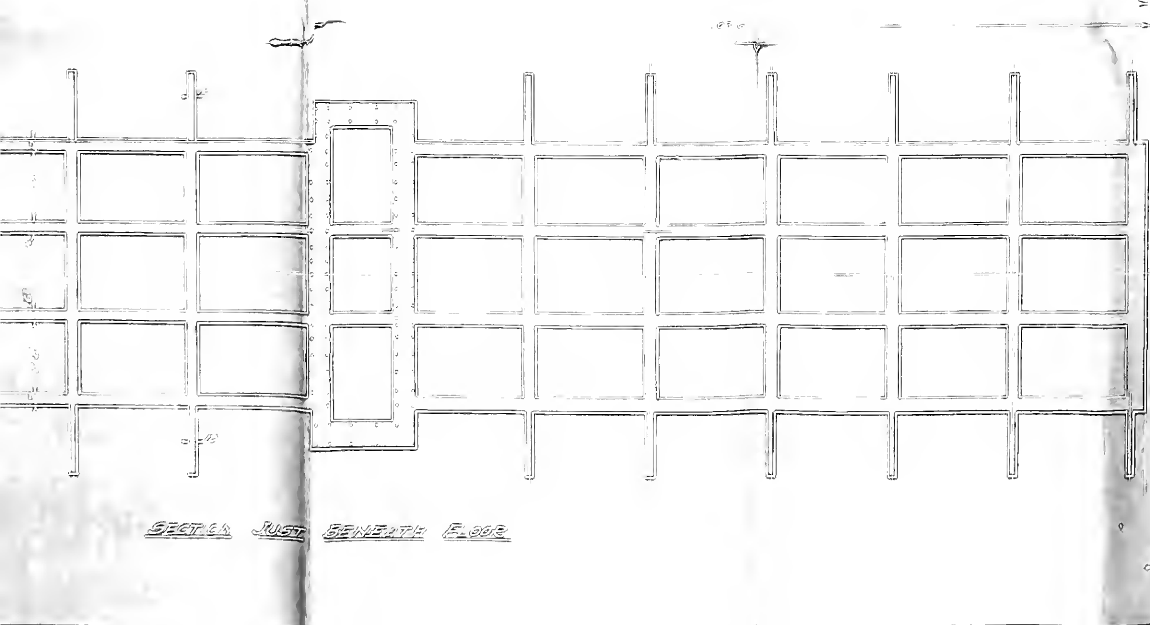
THESIS

OPEN SPANDREL REINFORCED CONCRETE ARCH BRIDGE
SPAN 210 FEET

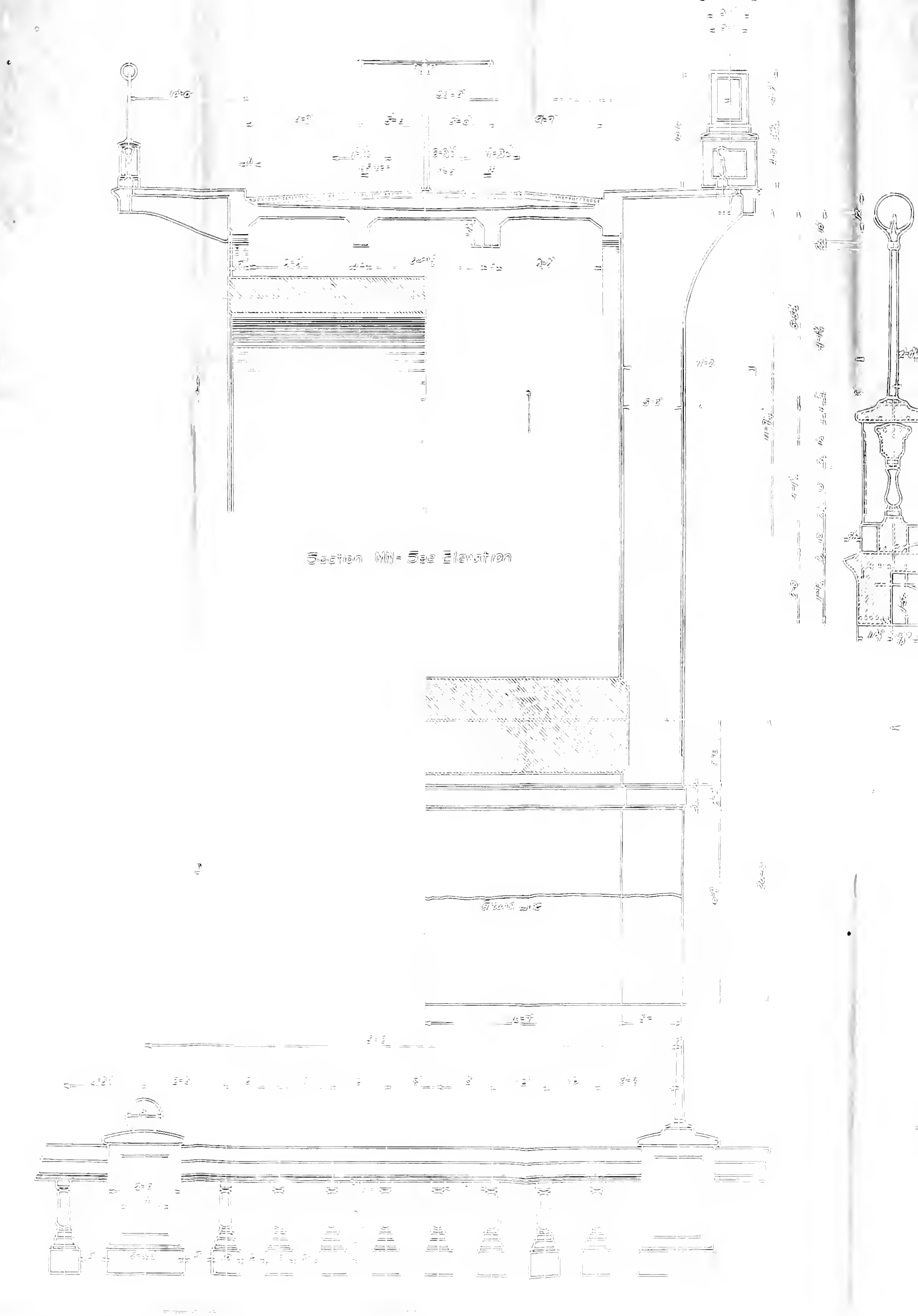
SCALE $\frac{1}{8}$ INCH = 1 FOOT MAY 1911

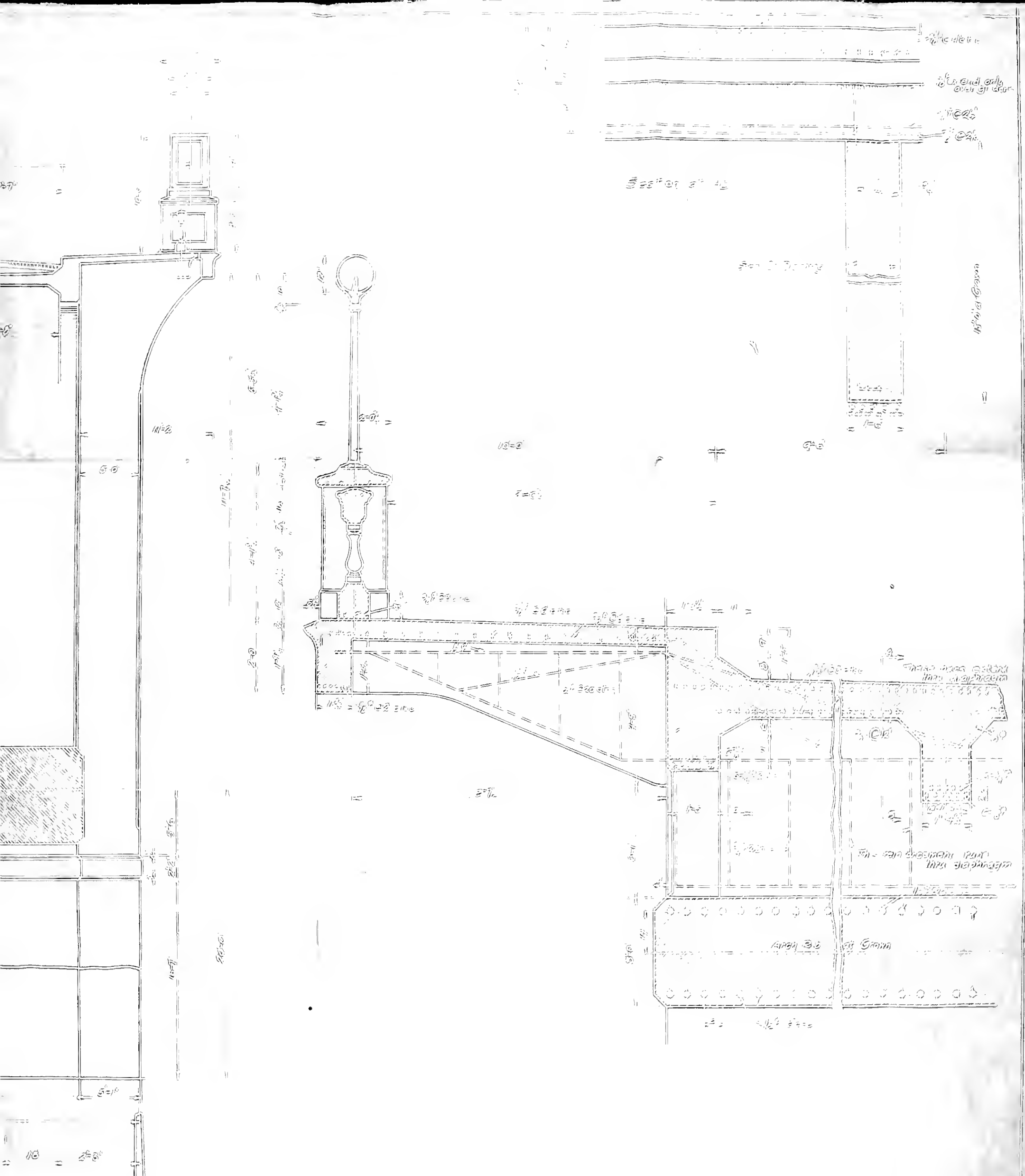


SECTION ALONG ℄ OF BRIDGE



SECTION JUST BENEATH FLOOR


Section MN - See Elevation



ARMOUR INSTITUTE OF TECHNOLOGY
CIVIL ENGINEERING DEPARTMENT

THESIS

OPEN SPANDREL REINFORCED CONCRETE ARCH BRIDGE

SPAN = 210'0" SCALES { 1/4" = 1 FOOT ; 3/4" = 1 FOOT } MAY - 1911 RSE 44-O

E.O. Mandler
C.H. Mary
R.J. Green